

CORPORATE STATISTICS II

GADE

TOPIC 1

EXERCISES

Exercise 1

In a certain company 10% of the employees get a promotion every year. If a sample of ten workers is selected randomly, determine:

- The random variable explaining the number of employees that will be promoted this year.
- The probability that 3 of the surveyed employees get a promotion.
- The probability that nobody in the sample get a promotion.
- The probability that more than 5 get a promotion.
- The probability that as many as 2 workers get a promotion.

Exercise 2

The number of cars entering a gas station hourly is, on average, 180. If the maximum number of cars that may be attended in such gas station is 10 per minute, find:

- The statistical model which best fits the behavior of the random variable.
- The probability that in a given minute arrive more cars than those which can be attended.

Exercise 3

The proportion of people with income over 60000 € a year in a city is 0.05%. If a sample of 5000 citizens is taken, obtain:

- The statistical model explaining the number of citizens enjoying such income.
- The probability of finding three people who receive that income.
- The probability that at least two citizens have that income.

Exercise 4

A recycling company knows that the amount of junk collected daily is below one ton. Calculate for a given day:

- The probability of accumulating as many as 600 kgs.
- The probability of collecting between 300 and 700 kgs.
- The expected amount of junk achieved, the variance and the standard deviation.

Exercise 5

The daily revenues, in thousands of euros, obtained by a shop follows a $N(5;1)$. Calculate:

- The probability of making between 4000 € and 6000 € in a day
- The probability of getting between 3000 € and 7000 € in a day
- The probability of making between 2000 € and 8000 € in a day
- The lowest revenue of the 10% of those days getting the highest income

Exercise 6

The monthly salary in euros earned by the workers of a company is a random variable $N(1300;100)$. Find:

- The probability that a given worker earns more than 1500 €
- The probability of an employee earning less than 1000 €
- The probability of an employee earning 1200 €
- The probability of an employee earning between 1100 and 1500 €
- The probability of an employee earning between 1200 and 1600 €

- f) The amount of money earned by the top 10% employees with the highest salary
- g) The amount of money earned by the bottom 20% employees with the lowest salary

Exercise 7

The Ministry of Public Works is going to put the build of a bridge out to tender among different construction companies. The price charged by the firms, in millions of euros, follows a $N(10;3)$. If the Ministry will choose the 30% of the projects with the lowest prices, what is the maximum budget for a company to get the contract?

Exercise 8

The manager at La Salud Hospital estimates that, on average, the number of patients entering the emergency unit is 5 per hour. Obtain:

- a) The statistical model justifying such behavior
- b) The variance of the random variable
- c) If the hospital releases hourly 5 beds for new patients coming from the emergency unit, what is the probability that in a given hour there is more patients than beds to attend them?

Exercise 9

“Pretty Woman” and “Espléndida” are two well-known brands in cosmetics keeping a close competition in sales. The perfumes sold monthly, in millions of dollars, by “Pretty Woman” follow a $N(50;10)$, while corresponding figures for “Espléndida” are better estimated by a $N(55;5)$. Assuming that a customer’s purchase is not affected by any other customer’s buy (independence), calculate:

- a) The probability for “Pretty Woman” selling more perfumes than “Espléndida” in a given month.
- b) The probability that the perfumes sold by “Espléndida” be higher than those sold by “Pretty Woman” in more than 3 million dollars at a given month.

Exercise 10

From past years, a teacher knows that the percentage of students passing an exam in Statistics is 0.4. Answer:

- a) If a random sample of 10 students is selected from a group, what is the probability of passing at least 8 students?
- b) If we consider that the students taking the exam are 300, what is the probability of passing at least 87?

Exercise 11

A manufacture sells between 20 tons and 40 tons of iron daily. What is the probability for selling more than 6250 tons in 200 days?

Exercise 12

Lasting Cars estimates that the time, in number of months, for an automobile of such a brand without having any serious breakdown is explained by a $N(36;2)$. Answer:

Calculate:

- a) The probability for a given car to have a breakdown in the first 32 months.

- b) Having called randomly 10 of their clients, what is the probability for at least two of them having their car broken in the first 32 months.
- c) Redo question b considering that the number of clients surveyed is 1000.

Exercise 13

An insurance company knows that the number of policyholders in the life area is 300000 and the probability of an insured dying in a given year is 0.003. Find:

- a) The number of claims expected in a given year
- b) The probability that in a given year the company has to face more than 950 claims.
- c) The maximum number of claims faced by the company in a given year with a probability of 90%.

Exercise 14

The life of a light bulb, in months, is modeled by a $N(20;2)$. Provided that the guarantee offered by the manufacturer allows for changing those light bulbs with life under 17, answer the following questions:

- a) What is the percentage of units that will be changed?
- b) If a simple random sample of ten units is selected, ¿what is the probability of changing at least one?
- c) Redo question b considering a sample size of 100.

Exercise 15

The number of phone calls received in a call centre is, on average, 120 per minute. Determine:

- a) The probability of receiving more than 3 phone calls per second.
- b) The probability of entering more than 160 phone calls per minute.
- c) The maximum number of phone calls entering in a minute with a probability of 99%.

Exercise 16

In a region a strange disease infects about 0.1% of the population. Answer:

- a) If a random sample of 10 people is selected, what is the probability that less than 3 be ill.
- b) Redo question a considering a random sample of 1000 people

Exercise 17

Solve the following cases where X follows a $N(0;1)$:

- a) $P(X \geq 0,56)$
- b) $P(X \leq -0,24)$
- c) $P(X \leq 1,36)$
- d) $P(X \geq -2,5)$
- e) $P(0,3 \leq X \leq 2,89)$
- f) $P(-0,7 \leq X \leq -0,15)$
- g) $P(-1,12 \leq X \leq 1,63)$
- h) $P(X \geq a) = 0,5675$ ¿a?

- i) $P(X \geq a) = 0,0192$ $\hat{a}?$
- j) $P(X \leq a) = 0,8485$ $\hat{a}?$
- k) $P(X \leq a) = 0,2389$ $\hat{a}?$
- l) $P(-0,68 \leq X \leq a) = 0,7289$ $\hat{a}?$

Exercise 18

Solve the following probabilities:

- a) $P(\chi^2_{20} \geq k) = 0,05$ $\hat{k}?$
- b) $P(\chi^2_{15} \geq k) = 0,02$ $\hat{k}?$
- c) $P(\chi^2_{16} \geq k_1) = 0,975;$ $\hat{k}_1?$ $P(\chi^2_{16} \geq k_2) = 0,025$ $\hat{k}_2?$
- d) $P(\chi^2_{16} \geq k_1) = 0,995;$ $\hat{k}_1?$ $P(\chi^2_{16} \geq k_2) = 0,005$ $\hat{k}_2?$

Exercise 19

Solve the following probabilities:

- a) $P(t_{25} \geq a) = 0,05;$ $\hat{a}?$
- b) $P(t_{25} \geq a) = 0,01;$ $\hat{a}?$
- c) $P(t_{20} \geq a) = 0,025;$ $\hat{a}?$
- d) $P(t_{20} \geq a) = 0,005;$ $\hat{a}?$

Exercise 20

Solve the following probabilities:

- a) $P(F_{20,15} \geq a) = 0,05;$ $\hat{a}?$
- b) $P(F_{10,30} \geq a) = 0,05;$ $\hat{a}?$
- c) $P(F_{30,20} \geq a) = 0,01;$ $\hat{a}?$
- d) $P(F_{15,10} \geq a) = 0,025;$ $\hat{a}?$

Solutions for the exercises 17-20:

17. $N(0;1)$:

- a) 0,2877
- b) 0,4052
- c) 0,9131
- d) 0,9938
- e) 0,3802
- f) 0,1984
- g) 0,817

h) -0,17

i) 2,07

j) 1,03

k) -0,71

l) 2

18. Chi-Squared

a) 31,41

b) 28,259

c) $k_1 = 6,908$ $k_2 = 28,845$

d) $k_1 = 5,142$ $k_2 = 34,267$

19. t-Student

a) 1,708

b) 2,485

c) 2,086

d) 2,845

20. F Fisher-Snedecor

a) 2,33

b) 2,16

c) 2,78

d) 3,52