1. A random variable follows a uniform distribution defined over the range [0;θ]. We want to estimate the unknown parameter θ selecting a s.r.s. of size n and using two possible estimators:

$$\theta_1^* = \overline{x} \qquad \theta_2^* = k\overline{x}$$

Determine:

a) The bias of the first estimator.
b) The value of *k* for the second estimator being unbiased.

2. Obtain the unbiased estimator of the population variance of certain variable, based on this s.r.s.: 60, 61, 63, 57, 60, 59, 62, 58, 60.

3. In a population N(μ;1) the unknown parameter μ is going to be estimated by two estimators based on a s.r.s. of size 2. These are those estimators:

$$\mu_1^* = \frac{2}{3}x_1 + \frac{1}{3}x_2 \qquad \mu_2^* = \frac{2}{5}x_1 + \frac{3}{5}x_2$$

Answer explaining your reasoning:

a) Are both unbiased?
b) Which one of them is more efficient?
c) Calculate the MSE.

4. Suppose we have a variable B(m;p) and have to choose between these two estimators based on bias, efficiency and consistency:

$$p_1^* = \frac{\overline{x}}{m} \qquad p_2^* = \frac{\overline{x}}{m+1}$$

Which one is better, having selected a s.r.s. of size n?

5. Consider a random variable N(μ,σ) and the following estimators of the unknown mean μ, having obtained a s.r.s. of size n:

$$\mu_1^* = \frac{x_1 + x_2 + ... + x_n}{n-2} \qquad \mu_2^* = \frac{x_1 + x_2 + ... + x_n}{n}$$

Which one would you prefer based on bias, efficiency and consistency?

6. The amount of vegetables sold by a greengrocer's, in kilograms, is a variable N(μ,1) being μ>3. If a s.r.s. of size 4 has been obtained, choose between these two estimators regarding the properties of bias and efficiency:

$$\mu_1 = \frac{1}{4}(x_1 + x_2 - x_3) \qquad \mu_2 = \frac{1}{2}(x_1 + x_2)$$

7. In a population B(1;p) study the bias, efficiency and consistency of the following estimators of the unknown parameter *p*, after taking a s.r.s. of size n=4:

$$p_1^* = x_1 \qquad p_2^* = \frac{2x_1 + 3x_3}{3}$$

8. Determine both the GMM and the MLE estimators of the unknown parameter *p* in a *B(1,p)* based on a s.r.s. of size n.

9. Obtain both the GMM and the MLE estimators of the unknown parameter λ in a *P(λ)* based on a s.r.s. of size n.

10. Determine both the GMM and the MLE estimators of the unknown parameter *p* in a *B(m,p)* based on a s.r.s. of size n.

11. Let ξ be a variable with the following probability distribution:

$$P(\xi = x) = \theta(1-\theta)^{x-1} \text{ para x = 0,1,2,... ; } \theta \geq 0 .$$

Obtain the MLE estimator of the unknown parameter *θ* based on a s.r.s. of size n.

12. Let ξ be a variable with the following density function:

$$f(x,\theta) = \frac{1}{\theta} e^{-x/\theta} \quad ; \quad x \geq 0 \quad \theta > 0$$
$$f(x,\theta) = 0 \quad ; \quad x < 0$$

Having selected a s.r.s. of size n, determine:

a) The MLE estimator of θ

b) The GMM estimator of θ

13. Let ξ a variable $N(\mu,\sigma)$. Having obtained a s.r.s. of size n, determine:

a) The MLE estimator of μ when σ is known

b) The MLE estimator of σ when μ is known

c) The MLE estimators of both μ and σ when they are unknown

d) The GMM estimators of both μ and σ when they are unknown

14. Let ξ be a variable with the following density function:

$$f(x,\theta) = \frac{2(\theta - x)}{\theta^2} \quad ; \quad 0 \leq x \leq \theta$$

Obtain the GMM estimator of the unknown parameter *θ* based on a s.r.s. of size n.

15. Consider a population featured by the following probability distribution:

$$P(\xi=-1)=\frac{1-\theta}{2} \qquad P(\xi=0)=\frac{1}{2} \qquad P(\xi=1)=\frac{\theta}{2} \qquad \text{with } 0<\theta<1$$

A s.r.s. of size 50 has been selected from that population, the results being:

x_i:	-1	0	1
n_i:	10	25	15

a) Obtain the MLE estimator of θ. Obtained the corresponding MLE estimate.

b) Obtain the GMM estimator of θ. Is that estimator unbiased? And consistent?

c) Obtain the corresponding GMM estimate.

16. The probability distribution of certain random variable ξ is defined as:

$$P(\xi=1)=\frac{1-\theta}{3}; \quad P(\xi=2)=\frac{\theta}{3}; \quad P(\xi=3)=\frac{2}{3}$$

A s.r.s. with 30 observations has been selected. These are the results:

x_i	1	2	3
n_i	5	5	20

Obtain the MLE estimation and the MLE estimate of θ associated to that sample.