Universidad Rey Juan Carlos

GADE BILINGÜE

CORPORATE STATISTICS II
Review of random variable

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UNIT I: STATISTICS AND INFORMATION

TOPIC 1: Introduction to Statistics

1. Statistical Sciences. The statistical method

STATISTICAL SCIENCES

1. Statistical Sciences. The statistical method

DESCRIPTIVE STATISTICS

❖ To collect:
- External sources
- Internal sources

❖ To organize:
- Frequency distributions
- Charts and graphs

DATA

THE POPULATION → THE SAMPLE

❖ To analyze:
- Measures of Central Tendency: a; Me; Mo
- Measures of Non Central Tendency: Qi; Di; Pi
- Measures of Variability: S^2; S; CV
- Measures of shape: skewness

1. Statistical Sciences. The statistical method

INFERENCE

- Sampling technics:
  - Simple Random Sampling
  - Stratified Random Sampling
  - Other

THE POPULATION

PARAMETER

←

ESTIMATION

THE SAMPLE

- Estimation technics:
  - Point estimators
    - Properties
    - Methods to obtain
  - Confidence interval estimation
  - Hypothesis testing

1. Statistical Sciences. The statistical method

PROBABILITY

- ❖ Concept:
  - Axiomatic definition
  - Methods of asigning probabilities
  - Conditional probability and independence

THE POPULATION

PARAMETER

ESTIMATION

THE SAMPLE

- ❖ Random variable: discrete and continuous
- ❖ Probability distribution
- ❖ Distribution function
- ❖ Density distribution function
- ❖ Moments of distributions
- ❖ Probability distributions: binomial, Poisson, Normal and other
- ❖ The Central Limit Theorem

UNIT III: PROBABILITY

TOPIC 5: Random variable. Probability distribution. Cumulative distribution function

5.1 Random variable: concept and classification

- Random variable (RV): it is a variable taking numerical values determined by the result of a re or a rp
- Sometimes the RV comes directly (example: the number of clientes entering a shop in one hour)
- But other times the basic events coming from a rp or a re are qualitative (examples: the marital status, the economic situation or flipping a coin into the air twice and observe the outcome)
- In these cases the RV arises from a function assigning a real number to each elementary event S_e in the sample space Ω:

$$\xi: \Omega \longrightarrow R$$
$$S_e \longrightarrow x \in R$$

- In a RV what is random is the event to happen not the number assigned to each basic event

5.1 Random variable: concept and classification

- A RV may be discrete or continuous
- A discret RV (DRV) is a RV taking a finite or countably infinite number of values (the size of a family, the number of phone calls received in an office in a given day, …)
- A continuous variable (CRV) is a RV taking an infinite and not countable number of values. In this case the values of the CRV are intervals in the real line R (the time elapsed between two phone calls in an office, the amount of money taken every day by a department store, the monthly family income in certain country, …).
- A CRV assumes that the variable can be measured with complete accuracy using infinite number of decimal places
- However most of the times we use seconds, cents or millimeters to measure a variable as if it were discrete.

5.2 Probability distribution

- For DRV:
- It is a function assigning probabilities to the basic outcomes of Ω, verifying Kolmogorov's Axioms:
    - $0 \leq P(x_i) \leq 1$
    - $\sum P(x_i) = 1$
- It may be defined by a mathematical formula, a table or a graph

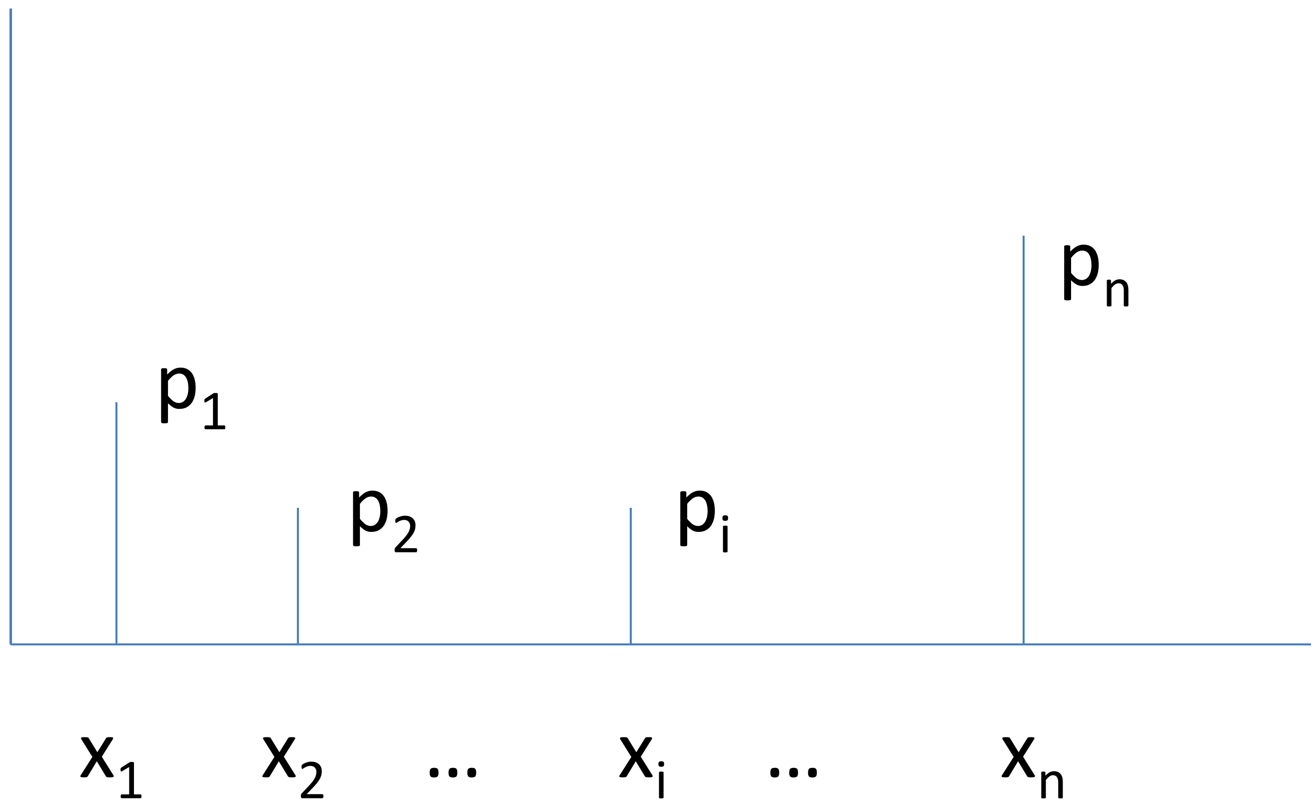
5.2 Probability distribution

Tabular definition

Values	Probability $P(\xi = x_i)$
x_1	$P(x_1)$
x_2	$P(x_2)$
….	….
x_i	$P(x_i)$
….	….
x_n	$P(x_n)$
	1

5.2 Probability distribution

Graphical definition



5.2 Probability distribution

- For CRV
  - A continuous probability distribution assigns probability to intervals
  - Hence: $P(x_i) = 0$; $i = 1..n$
  - The basic interval has an infinitesimal length dx
  - The probability in such basic interval is defined as follows:

$$f(x)\,dx$$

  - Where f(x) is called density function and represents density or concentration of probability for each x value
  - f(x) must verify two conditions:

$$1.\ f(x) \geq 0 \qquad 2.\ \int_{-\infty}^{+\infty} f(x)dx = 1$$

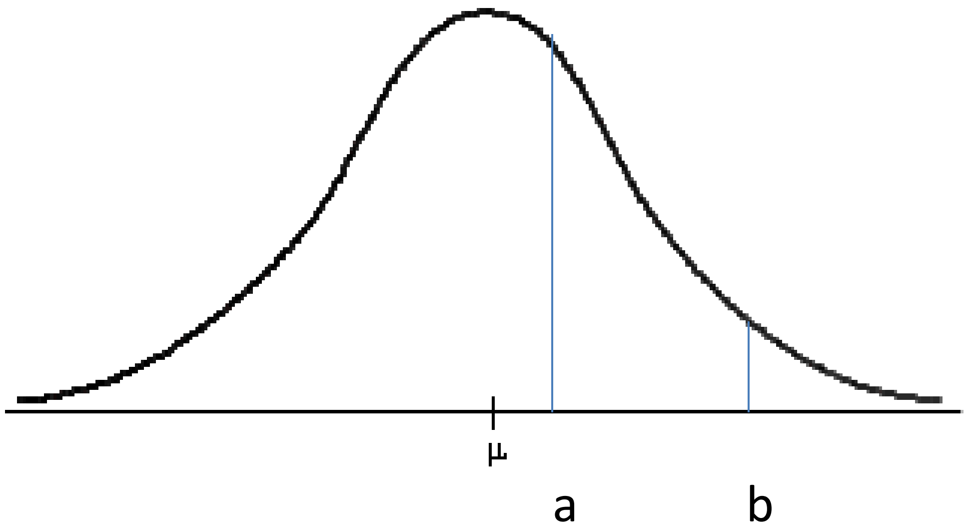
5.2 Probability distribution

- For CRV
  - It may be defined mathematically or graphically
  - Therefore the probability in a given interval [a;b] is calculated as:

$$\int_a^b f(x)dx$$

5.2 Probability distribution

- Graphically:



5.3 Cumulative distribution function F(x)

- It is the probability for a RV taking the value x or less:

$$F(x) = P(\xi \leq x)$$

  - Also:

$$F(x) = P(\xi \leq x) = P(-\infty < \xi \leq x) = P(\xi \in (-\infty;x])$$

  - Properties:

1. $F(+\infty) = P(\xi \leq +\infty) = P(\Omega) = 1$
2. $F(-\infty) = P(\xi \leq -\infty) = P(\emptyset) = 0$
3. F(x) is a not decreasing function:
   Si $x_1 \leq x_2 \longrightarrow F(x_1) \leq F(x_2)$
4. Si a < b then $P(a < \xi \leq b) = F(b) - F(a)$
5. In a CRV: $f(x) = F'(x)$

Bibliography of the topic

- Newbold, P., Carlson, W.L. and Thorne, B. (2010): «Statistics for Business and Economics». Pearson. Seventh Edition. Global Edition.
- Bernstein, S. and Bernstein, R. (1999): «Schaum's Outlines. Elements of Statistics I: Descriptive Statistics and Probability». Mc Graw Hill. USA.
- Montiel, A.M., Rius, F. y Barón, F.J. (1997). «Elementos Básicos de Estadística Económica y Empresarial». Prentice Hall.
- López de la Manzanara Barbero, J (2008). «Problemas de Estadística». Editorial Pirámide.
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UNIT III: PROBABILITY

TOPIC 6: Moments of a probability distribution

6.1 Raw moments. The expected value

- Raw moment of order r:

$$\mu_r = E(\xi^r)$$

  - In DRV:

$$\mu_r = \sum_{i=1}^{n} x_i^r \cdot P_i$$

  - In CRV:

$$\mu_r = \int_{-\infty}^{+\infty} x^r f(x)dx$$

  - The expected value of a RV: $\mu_1 = E(\xi) = \mu$

  - In DRV:

$$\mu_1 = \sum_{i=1}^{n} x_i \cdot P_i$$

  - In CRV:

$$\mu_1 = \mu = \int_{-\infty}^{+\infty} x f(x)dx$$

6.1 Raw moments. The expected value

- Properties of the expected value:

1. If k is a constant, then $E(k) = k$
2. $E(k*\xi) = k * E(\xi)$
3. $E(\xi_1 \pm \xi_2) = E(\xi_1) \pm E(\xi_2)$
4. If ξ_1 and ξ_2 are independent RV then
   $E(\xi_1 * \xi_2) = E(\xi_1) * E(\xi_2)$

6.2 Central moments. The variance

- Central moment of order r: $m_r = E(\xi - \mu)^r$
  - In DRV: $m_r = \sum_{i=1}^{n} (x_i - \mu)^r \cdot P_i$
  - In CRV: $m_r = \int_{-\infty}^{+\infty} (x-\mu)^r f(x)dx$
  - The variance of a RV:

$$m_2 = E(\xi - \mu)^2 \equiv \sigma^2 \equiv V(\xi) = E(\xi^2) - \mu^2 = \mu_2 - \mu^2$$

  - In DRV: $\sigma^2 = \sum_{i=1}^{n} (x_i - \mu)^2 \cdot P_i = \sum_{i=1}^{n} x_i^2 \cdot P_i - \mu^2$
  - In CRV: $\sigma^2 = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)dx = \int_{-\infty}^{+\infty} x^2 f(x)dx - \mu^2$

6.2 Central moments. The variance

- The variance. Properties:

1. $V(X) \geq 0$
2. $V(k) = 0$, where k is a constant
3. $V(k*X) = k^2*V(X)$
4. $V(a + bX) = b^2*V(X)$, where a and b are constants
5. $V(X_1 \pm X_2) = V(X_1) + V(X_2)$ only when X_1 and X_2 are independent

6.3 Other measures

- The standard deviation: $\sigma = \sqrt{V(X)}$
- Pearson's Coefficient of Variation: $CV = \dfrac{\sigma}{|\mu|}$
- Skewness coefficient: $\gamma_1 = \dfrac{\mu_3}{\sigma^3}$
- Standardization of a variable:

  Given a variable X with mean μ and variance σ^2, both known, then its standardized variable is obtained as follows:

$$X^* = \frac{X - \mu}{\sigma}$$

  The standardized variable verifies that E(X*) = 0 and V(X*) = 1

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