

1. A supermarket allocates 100 tins of three kind of drinks in a shelf: 20 of lemon, 30 of orange and 50 of cola. Due to an especial offer, consisting of buying the second unit at a half price, the customers buy the bottles in twos. Each bottle has the same probability of being selected and each bottle sold is replaced immediately. If any pair of bottles sold are considered a sample with  $n=2$ , then calculate (for practical concerns, assign the number 1 to lemon taste, the number 2 to orange taste and the number 3 to cola flavour):

- The probability distribution of the first element in the sample
- The probability distribution of the second element in the sample
- The joint probability of the sample
- The sampling distribution of the sample mean (including values for its expected value and its variance)
- The sampling distribution of the sample variance (including values for its expected value and its variance)

2. Let  $\xi$  be a variable following a  $B(1;p)$ . Then obtain the joint probability of the sample for s.r.s. of size  $n=3$ , apart from the sampling distribution of the sample mean.

3. Let  $\xi$  be a variable following a  $N(\mu;\sigma)$ . Then obtain the joint probability of the sample for s.r.s. of size  $n$ , apart from the sampling distribution of the sample mean.

4. Given a population  $\xi$  with the following probability distribution:

$x_i$	-1	1	2
$P_i$	0.25	0.5	0.25

Calculate, taking all the possible s.r.s. of size  $n=2$ :

- The probability distribution of each element in the sample
- The joint probability distribution of the sample
- The sampling distribution of the sample mean (including values for its expected value and its variance)
- The sampling distribution of the sample variance (including values for its expected value and its variance)

5. The daily production of soft-drinks at a firm, in thousands of liters, is given by the following random variable:

$$f(x) = \begin{cases} \frac{1}{5} & 0 \leq x \leq 5 \\ 0 & \text{other} \end{cases}$$

Determine, explaining your reasoning:

- a) The sampling distribution of the sample mean, selecting a s.r.s. of size  $n=200$ , including its expected value and its variance, interpreting the results.
- b) The probability for the sample mean in the previous question being lower than 2000 liters.

6. Given a normally distributed population with mean of 1 and standard deviation of 4, a s.r.s. of size 16 is obtained. Then calculate:

$$P(\bar{x} > 3) \quad P(0.5 < \bar{x} \leq 1.7)$$

7. A random variable  $\xi$  is characterized by an expected value of 32 and a variance of 220. Provided that a s.r.s. of size  $n=100$  is selected, calculate the probability for the sample mean taking values between 30.56 and 33.1.

8. Temperature in Valladolid in Celsius at a given time in the morning, along January, follows a normal distribution with  $\mu=0.5$  and standard deviation unknown. Having selected a s.r.s. of size  $n=5$  with these results:

$$1 \quad -2,3 \quad 1,7 \quad 2,1 \quad -0,67$$

Obtain  $P(\bar{x} \geq 0.7)$

9. It has been estimated that 3 out of 5 people in a group are women. If a s.r.s. of size  $n=40$  is selected from that group, determine the probability for the proportion of women falling between 55% and 65%.

10. Myfriend is a corporation whose anual statements are being audited. On these grounds, certain manager at the Human Resources Department has been asked about the salary earned by their workers, being the answer a normally distributed random variable with mean of 2000 € and standard deviation of 500 €. Then the auditor checked the payroll and obtained a s.r.s. of 100, observing then the average salary was 1850 €. What will be the auditor's opinion on the information given by the manager? Explain your reasoning using statistical procedures.

11. In certain corporation employees have been asked about their abilities in English language, 20 per cent of them affirming proficiency on that matter. However, the academy English-up has run an exam to 50 employees selected randomly (s.r.s.), observing that only 3 of them master the language. What can be said about employees' opinion regarding their knowledge in English? Explain your reasoning using statistical procedures.

12. The daily demand of certain product varies between 200 and 800 units. The company manufacturing it achieves profits, in a given year, whenever the daily demand along that period is, on average, above 520 units. Determine the probability for the firm getting profits in a given year.

13. Let  $X$  be a population, whose probability distribution is  $N(1.7;4)$ . Provided that a s.r.s. of size 10 has been selected, obtain the value “ $k$ ” verifying that:

$$P(s > k) = 0.99$$

Where  $s$  stands for sampling standard deviation.

14. The fabrication rules of certain electric component establish its life having a variability lower than 40 hours, measured by the standard deviation. Considering that the random variable follows a normal distribution, the department of quality at the company is going to obtain a s.r.s. of 30 components and calculate the sample variance. Determine the maximum value for the sample variance verifying the fabrication rules in that sample with a probability of 95%.

15. ALBASA and BUENSA are two leader companies in the distribution economic sector in a given region. Certain manager of one of those firms assures that sales of both companies are equal ( $\mu_x = \mu_y$ ) and so is their variability ( $\sigma_x = \sigma_y$ ). Assuming that sales are normally distributed, that manager has selected two independent samples (s.r.s.) with the sales of each company in 34 days. These have been the results ( $X$  for ALBASA and  $Y$  for BUENSA):

$$\sum x_i = 191 \quad \sum x_i^2 = 1379 \quad \sum y_i = 182 \quad \sum y_i^2 = 1158$$

Is the manager right? Explain your reasoning using statistical procedures.