Surnames:	***Name:***

Weights: 40% for section A and 60% for section B

SECTION A: MULTIPLE CHOICE

Maximum grade: 10 points
A correct answer adds 1 point
A wrong answer substracts 0.2 points
Minimum grade for assessing SECTION B: 4 points
Those answers not marked in the mask will not be considered

1.- Choose the right option regarding probabilistic sampling:
a) It allows to measure the error made in the inference
b) It is subjuctive
c) It always produces biased samples
d) Snowball sampling is one of those techniques

2.- A call centre receives an average of 2 telephone calls per second. Then a s.r.s. of 40 seconds has been obtained. Select the right choice regarding each element in the sample:
a) It follows a $N(\mu;\sigma)$
b) It follows a $B(1;p)$
c) It follows a $U[a;b]$
d) It follows a $P(\lambda)$

3.- Let X be a random variable with mean μ and variance σ^2. Then a s.r.s. of size n has been obtained with two point estimators of μ being under consideration:

$$\hat{\mu}_1 = 2x_1 - x_2 \qquad \hat{\mu}_2 = \bar{x}$$

Choose the right option:
a) Both estimators are biased
b) Both estimators are unbiased
c) The first estimator is unbiased and the second one is biased
d) The first estimator is biased and the second one is unbiased

4.- Choose the right option regarding the sampling variance s^2:
a) It is an unbiased estimator of μ
b) It is the maximum likelihood estimator of σ^2 in a $N(\mu;\sigma)$
c) It follows a Student's t distribution
d) It is an unbiased estimator of σ^2

5.- Choose the right option regarding Inference:
a) It is a technique which employs samples in order to characterise populations
b) It is a technique which employs populations in order to characterise samples
c) It is a technique which employs parameters in order to characterise estimators
d) It is a technique which employs parameters in order to characterise statistics

6.- Choose the right option regarding a confidence interval estimation over the mean of a $N(\mu;\sigma)$:
a) The higher the sample size the lower the level of confidence
b) The lower the sample size the higher the level of confidence
c) The lower the sample size the higher the precision of the estimation
d) The higher the dispersion of the variable the lower the precision of the estimation

7.- Choose the right option regarding a hypothesis test:
a) We know and control the probability of rejecting the null hypothesis being true
b) We know and control the probability of rejecting the null hypothesis being false
c) We know and control the probability of accepting the null hypothesis being false
d) We do not know and cannot control any probability in a hypothesis test

8.- Let X and Y be two independent variables being distributed as follows:

$$X \sim N(2;\sigma) \qquad Y \sim N(5;\sigma)$$

And let M be a variable defined in this way:

$$M = \frac{X - 2Y}{2}$$

Choose the right option regarding the distribution of M:
a) $N(-4;\sigma\sqrt{1.2})$
b) $N(6;\sigma\sqrt{4.3})$
c) $N(-4;\sigma\sqrt{0.75})$
d) All the previous answers are wrong

9.- Choose the right option regarding the p-value in a hypothesis test:
a) It depends on β
b) It depends on α
c) It depends on the test statistic
d) It depends on the critical value

10.- A 95% confidence interval estimation for the quotient of variances of two populations $\frac{\sigma_x^2}{\sigma_y^2}$ is given by [0.25;0.6]. Choose the right option:
a) There are not significative differences between the variances of both populations
b) The variance of X being twice the variance of Y is a possible solution
c) The variance of Y being twice the variance of X is a possible solution
d) It cannot be said anything regarding the relationship between the respective variances

(model x) **MARK YOUR ANSWER WITH AN X**

	A	**B**	**C**	**D**
Question 1	**X**			
Question 2				**X**
Question 3		**X**		
Question 4		**X**		
Question 5	**X**			
Question 6				**X**
Question 7	**X**			
Question 8	**X**			
Question 9			**X**	
Question 10			**X**	

SECTION B: Exercise
Maximum grade: 10 points
Minimum grade to pass: 5 points

Exercise 1 (4 Points)

An audit firm is checking certain company. A manager working in the Human Resources Department of the company assures that the average net monthly wage, in thousands of euros, earned by the employees is higher than 1.5. In order to test that hypothesis the audit firm has obtained a s.r.s. of 16 workers and asked them for their monthly wages. This variable is assumed to follow a N(μ;σ). The results coming from the sample are:

$$\sum x_i = 28.57 \qquad \sum x_i^2 = 52.78$$

a) Run the corresponding test at a 5% significance level formulating the hypothesis, the test statistic, the critical region and the decision made. Also solve using the *p-value*.

b) Besides that, the audit firm wishes to verify if the standard deviation is equal to 0.3. Run the corresponding test at a 5% significance level formulating the hypothesis, the test statistic, the critical region and the decision made. Also solve using the *p-value*.

c) Discuss the possible errors made (type I and type II) based on the decision made in each one of the previous tests.

Exercise 2 (4 points)

In the context of the previous exercise the audit firm wants to know the proportion *p* of employees doing overtime regularly. With that purpose a s.r.s. of *n* workers will be selected (each element being a 1 for a "yes" situation and a 0 for a "no" case).

a) Which one of the following estimators of *p* is the best?

$$p_1^* = \frac{2x_1 - x_2}{3} \qquad\qquad p_2^* = \bar{x}$$

Justify using this properties: unbiasedness, efficiency and consistency

b) How many workers must be selected in order to achieve a sampling error of 0.03 when building a 95% confidence level interval estimation of *p*? (Consider maximum uncertainty)

c) Finally a s.r.s. of 200 workers has been obtained finding 120 of them doing overtime regularly. Elaborate the 95% confidence level interval estimation for the population proportion. What is the precision achieved? Why is it different than that existing in question b)?

Exercise 3 (2 Points)

Run a Shapiro-Wilks goodness of fit test with $\alpha = 1\%$ in order to check if the s.r.s. of wages obtained in exercise 1 matches a $N(\mu;\sigma)$. Formulate the hypothesis, the test statistic, the critical region and the decision made. Also solve using the *p-value*. The corresponding complete ordered sample is:

0,9358	1,4307	1,4438	1,4641	1,6957	1,7137	1,8006	1,8070
1,8475	1,8628	1,9480	1,9893	1,9962	2,0892	2,2255	2,3181